

AIAA 98-2882

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20th AIAA Advanced Measurement and Ground Testing Technology Conference

June 15-18, 1998 / Albuquerque, NM

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THE USE OF HEAVY GAS FOR INCREASED REYNOLDS NUMBERS IN TRANSONIC WIND TUNNELS

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Abstract

The use of a high molecular weight test gas to increase the Reynolds number range of transonic wind tunnels is explored. Modifications to a small transonic wind tunnel are described and the real gas properties of the example heavy gas (sulfur hexafluoride) are discussed. Sulfur hexafluoride is shown to increase the test Reynolds number by a factor of more than 2 over air at the same Mach number. Experimental and computational pressure distributions on an advanced supercritical airfoil configuration at Mach 0.7 in both sulfur hexafluoride and nitrogen are presented. Transonic similarity theory is shown to be partially successful in transforming the heavy gas results to equivalent nitrogen (air) results, provided the correct definition of gamma is used.

Introduction

The shortfall in Reynolds number in current U. S. transonic ground test facilities is well known (Mack, et al.¹), and with the advent of the proposed, so-called "megaliners" (600 to 800 passenger transonic, transport aircraft) this shortfall is likely to grow significantly in the coming years. Test data needed by the designers of these new large-scale aircraft will require new, higher Reynolds number test facilities, and recent estimates indicate that large-scale, high Reynolds number, transonic wind tunnels will be extremely expensive to build and operate. All of this is occurring in an era of increased commercial competition to reduce the design cycle time and lower the cost of developing new transport aircraft, and in a time of declining research budgets. The only facilities currently

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capable of producing the required flight-level Reynolds numbers at transonic speeds are cryogenic tunnels. Two such tunnels in the U.S., and one in Europe are available for commercial testing, but wind tunnel testing at 100 degrees Kelvin is far from routine and requires special care with regard to instrumentation and model construction. These cryogenic facilities rely on a combination of increased pressure and reduced temperature to increase the density and reduce the viscosity of the test gas (nitrogen). This technique can produce freestream unit Reynolds numbers in excess of 100 million per foot and, augmented by the additional benefit that the lower speed of sound at cryogenic temperatures results in reduced drive horsepower and model loads, offers the only currently viable solution for high Reynolds number testing. However, the cost and difficulty associated with cryogenic testing, and the unlikely possibility of constructing additional cryogenic facilities to relieve the crowded test schedules has initiated research at NASA into alternate techniques for increasing test Reynolds numbers.

One technique that may hold promise is that of increasing the Reynolds number through the use of a high molecular weight test gas. The advantages of such a technique are that testing can be done at ambient temperatures, and existing tunnels can be retrofitted to use "heavy gas". This latter advantage becomes even more significant in view of the high cost of constructing new facilities.

The use of heavy gas to increase Reynolds number is an old idea dating back to work by Huber², Schwartzberg³, vonDoenhoff, et al.⁴, and Chapman⁵. Chapman's work was an exhaustive examination of candidate gases that could be combined to produce gas mixtures with a ratio of specific heats (γ) near 1.4. Later work by Pozniak⁶, Treon, et al⁷, Yates and Sandford⁸, and others focused on relating results in gases where gamma did not equal 1.4 to equivalent results in air. Results from these earlier studies were

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inconclusive, and the technique has lain essentially dormant for the past 20 years. However, in 1991, Anderson⁹ conducted a computational study of transonic airfoils in sulfur hexafluoride and found that transonic similarity scaling could be used to relate results in SF₆ to equivalent air results for primarily inviscid flows, as long as the proper definition of gamma was used. Anderson also found that as viscous effects became more pronounced, and as the flow became more compressible, the scaling was less effective. Later, Bonhaus, et al.¹⁰ applied Anderson's code to two multi-element high-lift configurations in SF₆ and concluded that transonic similarity scaling was adequate for such flows.

As a result of Anderson's work, NASA began a program in 1991 to convert a small transonic wind tunnel to SF₆ operation in order to provide some experimental confirmation of Anderson's results, and to further define the range of application of transonic similarity theory to heavy gas flows. A progress report on this program has been given by Anders¹¹. The current paper will discuss the results of an initial study of a supercritical airfoil at transonic speeds in SF₆ compared with nitrogen results using transonic similarity scaling. Computational results for both SF₆ and air will be compared to the experimental data. For the purposes of this paper air and nitrogen are assumed to be the same gas.

Methods

Experimental Facility

The test facility used for the current study is the 0.3-Meter Transonic Cryogenic Tunnel at the Langley Research Center. This facility, described in some detail in Mineck and Hill12, was used to develop the cryogenic test concepts leading to NASA's National Transonic Facility (NTF). The facility operates at pressures up to 6 atmospheres, temperatures from 100° to 300° K, and Mach numbers from 0.15 to 1.0. The test section is approximately 13" x 13" and the upper and lower walls are adaptive. The modifications required for operation with SF6 included a gas reclamation unit for charging and reclaiming the test gas, a gas analysis unit for real-time monitoring of gas composition, a gas warning system for personnel safety, and a designed specially heat exchanger. These modifications are discussed fully in Anders¹¹, and with the exception of the custom-designed heat exchanger, all of these units are off-the-shelf, commercially available systems. Figure 1 shows a

schematic of the modified tunnel with the additional systems required for heavy gas operation denoted.

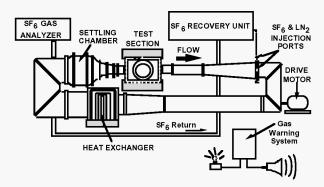


Fig. 1. Schematic of 0.3-Meter Transonic Cryogenic Tunnel with modifications for heavy gas operation.

Figure 2 shows the operating envelope of the tunnel. Although the Reynolds number range for SF_6 is approximately half that of cryogenic nitrogen, the upper limit approaches $60 \times 10^6 / \mathrm{ft.}$, substantially greater than for ambient temperature air operation.

Model

The model used for the current study was an advanced supercritical airfoil with 78 pressure taps over the upper and lower surfaces, mounted between sidewall turntables, and spanning the test section.

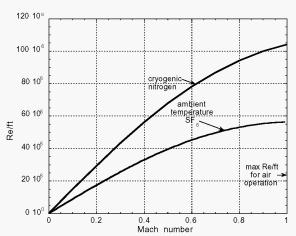


Fig. 2. Operating envelope for 0.3-Meter Transonic Cryogenic Tunnel.

The model was tested over a fairly limited angle-of-attack range (from +1.0 degrees to -1.0 degrees), and the upper and lower tunnel walls were contoured to minimize tunnel wall interference effects¹². The majority of the data were taken in the Mach number range from 0.70 to 0.72. The Reynolds

number based on airfoil chord was 30 x10⁶, although a limited amount of data was obtained at 15×10^6 .

The accuracy of the measured static airfoil pressures is estimated as $\pm 0.5\%$. The accuracy of the guoted Mach number is ± 0.002 , and the accuracy of the angle-of-attack settings is ± 0.05 degrees. Figure 3 shows an example of the repeatability of the airfoil surface pressure measurements in SF₆.

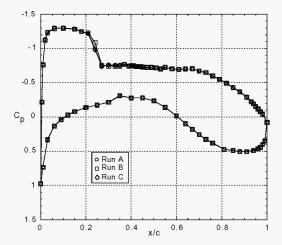


Fig. 3 Repeatability of SF₆ pressure distributions, $Re_c = 30 \times 10^6$, M = 0.72, $\alpha = 1.0$ deg.

Characteristics of Sulfur Hexafluoride

Sulfur hexaflouride (SF₆) is a colorless, odorless, non-toxic gas with a molecular weight of 146. It's principal commercial use is as a dielectric in high-voltage switchgear for arc suppression, and it is readily available from a number of manufacturers. It is essentially inert at laboratory temperatures, and, although it has no ozone depletion potential, it is a greenhouse gas. SF₆ is both calorically and thermally imperfect, but its properties are well-known and Equipment documented. for handling (condensers, vaporizers, storage tanks, pumps, etc.) are manufactured commercially for power companies and are readily available. The principal hazard from this gas is that of an asphyxiant since it displaces air.

The equation of state^{9,13} is well-represented

$$p = \frac{RT}{v-d} + \sum_{i=2}^{5} \frac{a_i + b_i T + c_i e^{-kT}/T_c}{(v-d)^i}$$

where R = specific gas constant and the various constants are given as: $a_2 = -49.9051433 \text{ N-m}^4/\text{kg}^2$ $b_2 = 5.485495 \times 10^{-2} \text{ N-m}^4/\text{kg}^2 - \text{ K}$ $c_2 = -2.375924505 \times 10^3 \text{ N-m}^4/\text{kg}^2$ $a_3 = 4.124606 \times 10^{-2} \text{ N-m}^7/\text{kg}^3$ $b_3 = -3.340088 x 10^{-5} \text{ N-m}^7/\text{kg}^3 - \text{K}$ $c_3 = 2.819595 \text{ N-m}^7/\text{kg}^3$ $a_4 = -1.612953 x 10^{-5} N - m^{10}/kg^4$ $a_5 = \text{-}4.899779 \text{x} 10^{\text{-}11} \text{ N-m}^{\text{13}} \text{/kg}^5$ $b_5 = 1.094195 \times 10^{-11} \text{ N-m}^{13}/\text{kg}^5 - \text{ K}$ $c_5 = -3.082731 \times 10^{-7} \text{ N-m}^{13}/\text{kg}^5$ k = 6.883022 $T_e = 318.8 \text{ K}$

 $d = 3.27367367 \times 10^4 \text{ m}^3/\text{kg}$

Anders¹¹ discusses the real gas behavior of SF₆ and concludes that the thermal imperfections of the gas, while not large at typical wind tunnel stagnation pressures, are, nonetheless, significant. Also, the caloric imperfections are shown to be quite large, and the variable specific heats must be accounted for in calculating the isentropic flow properties. In addition, Anders11 concludes that small amounts (less than 10%) of air and water vapor contamination of the test gas have little or no effect on the free-stream flow properties, but do effect the mixture viscosity slightly. In the present experiment the test gas composition was continuously monitored during tunnel runs and indicated an SF₆ purity of approximately 98%. Jenkins¹⁴ has developed a computer code to calculate the isentropic flow properties of SF₆ and this code was used for the data reduction of all the experimental results presented here.

Assuming for the moment that SF₆ is a perfect gas, it is easy to show that the Reynolds number increase for SF₆ over air at the same Mach number and total pressure can be written as:5

$$\frac{\operatorname{Re}_{hg}}{\operatorname{Re}_{air}} = \left(\frac{\mu_{air}}{\mu_{hg}}\right) \sqrt{\frac{\gamma_{hg}(MWt)_{hg}}{\gamma_{air}(MWt)_{air}}} \approx 2.4$$

where MWt = the molecular weight. Similar expressions can be developed for the dynamic pressure and the required drive horsepower:

$$\frac{q_{hg}}{q_{air}} = \left(\frac{\mu_{hg}}{\mu_{air}}\right) \sqrt{\frac{(MWt)_{air}}{(MWt)_{hg}}} \approx 0.4$$

$$\frac{HP_{hg}}{HP_{air}} = \left(\frac{\mu_{hg}}{\mu_{air}}\right) \left(\frac{(MWt)_{air}}{(MWt)_{hg}}\right) \approx 0.2$$

Although sulfur hexafluoride was one of the candidate gases originally proposed by Chapman, new refrigerant gases such as R134A (the replacement gas for Freon-12) have become common since that time and may, in fact, be preferable due to environmental (greenhouse) and cost concerns. However, SF₆ is representative of the class of high molecular weight gases that can, as shown above, increase the Reynolds number and decrease the dynamic pressure and drive horsepower. These characteristics may be particularly attractive for selected, existing transonic wind tunnels to provide an enhanced Reynolds number test range. Table 1 compares some of the physical properties of SF₆ with air and with two of the new refrigerant gases. Technically, the ratio of specific heats (γ) is not a constant for the three heavy gases listed in Table 1, but for the purpose of estimating the Reynolds number increase, it is treated as such. For all other purposes in this paper, the ratio of specific heats is treated as a variable.

gas	Molec. Wt.	γ	* μ	$\frac{Re_{\text{heavy gas}}}{Re_{\text{air}}}$
air	29	1.4	38.6	1.0
SF ₆	146	1.095	32.6	2.4
R134A	102	1.105	28.6	2.3
R125	120	1.109	31.3	2.3

* μ in 1b_f sec/ft²x 10⁸

Table 1. Gas characteristics.

Results and Discussion

Experimental Results

Typical pressure distributions over the test supercritical airfoil in both SF_6 and cryogenic N_2 are shown in figures 4a and 4b. There are clear differences in the pressure distributions between the two gases, especially for the cases at higher angles-of-attack where a shock forms on the forward part of the airfoil upper surface. The rear of the upper and lower surfaces show little effect from either gas or angle-of-attack.

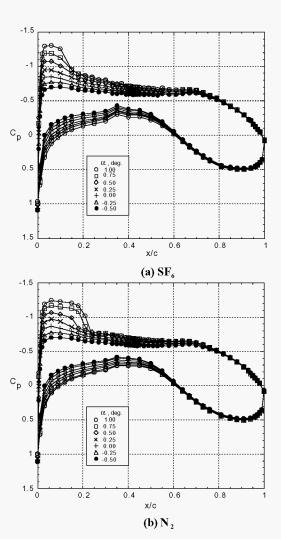
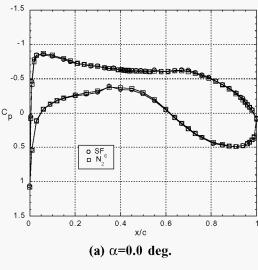
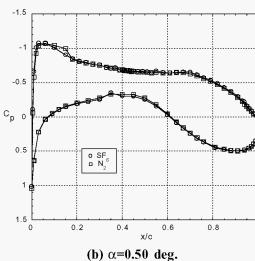


Fig. 4. Pressure distributions on test airfoil in sulfur hexafluoride and nitrogen, M=0.70, $Re_c=30 \times 10^6$.

Figures 5a, b, and c show a direct comparison between the two gases at three angles-ofattack. The agreement on the lower surface and on the rear of the upper surface, where the local Mach numbers are low, is fairly good, as would be expected for a nearly incompressible flow. However, on the forward part of the upper surface, as angle-ofattack increases and compressibility effects become larger (the local Mach number upstream of the shock for nitrogen reaches a maximum of 1.3), there are significant differences, especially in the region of the Obviously, thermodynamic shock. differences between the two gases become important for compressible flow. The shock location on the airfoil in nitrogen always appears downstream of the location in SF₆ at the same Mach number.





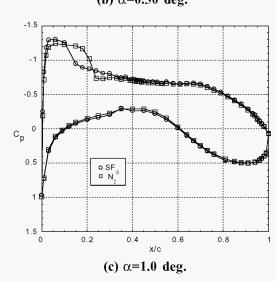
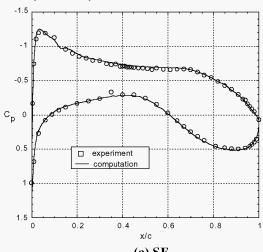


Fig. 5. Comparison of N_2 and SF_6 pressure distributions at M = 0.70, $Re_c = 30 \times 10^6$.

Computational Method and Results

The computations have been done using the unstructured grid Navier Stokes solver15 with suitable modifications to properly account for the non-ideal gas behavior of SF₆. Figure 6 shows examples of the computational results at $Re_c = 30 \times 10^6$ and $\alpha = 0.75$ degrees compared with the measured pressures for both SF₆ and N₂. The agreement between computation and experiment is considered to be quite good except for a consistent overprediction of the upper surface pressures downstream of the shock. It should be pointed out that the computation results in Fig. 6b are actually for air rather than nitrogen. This was done as a matter of convenience since for all practical purposes the two gases aerodynamically identical.



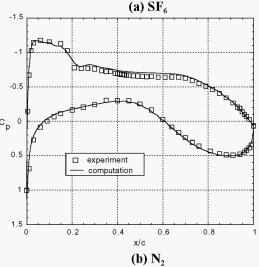


Fig. 6. CFD comparison with experimental data, Re_c =30x10⁶, M = 0.70, α =0.75 deg.

The agreement shown in figure 6a gives some assurance that the real gas behavior of SF_6 is correctly captured in the computations

Transonic Similarity

According to inviscid, small disturbance transonic similarity theory¹⁶, the flows between air and SF₆ will be equivalent provided that the similarity parameters are equal.

$$\left[\frac{1-M_{\infty}^{2}}{\left[\tau M_{\infty}^{2}(\gamma'+1)\right]^{\frac{2}{3}}}\right]_{heavy\ gas} = \left[\frac{1-M_{\infty}^{2}}{\left[\tau M_{\infty}^{2}(\gamma+1)\right]^{\frac{2}{3}}}\right]_{nitrogen}$$

where $\tau = t/c = max$. thickness/airfoil chord

For a given profile where the thickness, τ , is fixed, an appropriate definition of γ may be used with the above equation to determine a freestream Mach number in SF₆ so that the similarity parameter will match that of air. After the fact, the lift and pressure coefficients can be corrected by:

$$C_{l,nitrogen} = A C_{l,heavy gas}$$

$$C_{p,nitrogen} = A C_{p,heavy gas}$$

and the parameter A is given by

$$A = \left(\frac{\gamma' + 1}{\gamma + 1}\right) \left[\frac{M_{heavygas}^2}{M_{nitrogen}^2}\right] \left[\frac{1 - M_{nitrogen}^2}{1 - M_{heavygas}^2}\right]$$

Anderson⁹ demonstrated computationally that inviscid flows in SF₆ can be scaled to give excellent agreement with air provided that the proper definition of gamma is used. This is given by⁹

$$\gamma' = 2\Gamma_{\infty} - 1 = 1 + \left(\frac{\delta a^2}{\delta h}\right)_{S,\infty}$$

where a = speed of sound and h = enthalpy.

At a given freestream Mach number in air, the Mach number in SF_6 is determined by first using the freestream pressure and temperature in SF_6 to determine γ' . The new Mach number can then be determined using the similarity parameter given above.

Using the above definition of γ' in conjunction with the transonic similarity laws, inviscid computational results obtained in SF₆ have been demonstrated to consistently scale to yield excellent agreement with computations in air over a wide range of pressures and temperatures⁹. However, application of these scaling laws for computed viscous flows has been only partially successful.

Scaling Applied to Experimental Results

Typical transonic similarity scaling variables for the current experiment are given by:

$$M_{N2} = 0.70$$
 $M_{SF6} = 0.72$ $\gamma = 1.04$ $A = 0.967$

Using the similarity scaling laws just described, the nitrogen data at M=0.70 is compared in Figures 7a, b, and c with SF₆ data at the similarity Mach number of M = 0.72. The C_p values have been scaled using the parameter A defined above. As the figure illustrates, the comparison is good over most of the airfoil, with some differences on the upper surface, especially in the vicinity of the shock at the highest angle-of-attack. Comparisons at Re_e=15 x 10⁶ (not shown) indicate a similar difference in the region of the shock. In fact, for all cases in the current experiment where a shock was present, the shock location in SF₆ (at the similarity Mach number) was always slightly downstream of the shock location in N₂. This difference was so consistent in the experiment that it can not be attributed to experimental inaccuracy. It is likely that there are differences in the viscous shock-boundary layer interaction process between the two gases, and evidence of this can be found in the following comparisons with computations.

Figures 8a and 8b are plots showing SF₆ and N2 experimental results at the similarity Mach number, along with computational results for air and SF₆, again at the similarity Mach number. The vertical scale has been expanded to illustrate that the computations show exactly the same trend in the region of the shock as the experimental data. That is, the SF₆ calculations at the similarity Mach number predict a shock location slightly downstream of the shock location for air (nitrogen in the case of the experimental data). Also, note that downstream of the shock the pressure coefficients are slightly more negative for SF₆ than for air (nitrogen) in both the computations and the experiment. The reasons for these differences are not entirely clear, but Anderson⁹ noted this same effect in his earlier computational study and indicated that the SF₆ boundary layer was appreciably thinner than the air boundary layer, which could result in less upstream movement of the shock due to displacement effects. A few comments in the next section on viscous similarity may shed additional light on some of the viscous mechanisms that may be responsible for these differences.

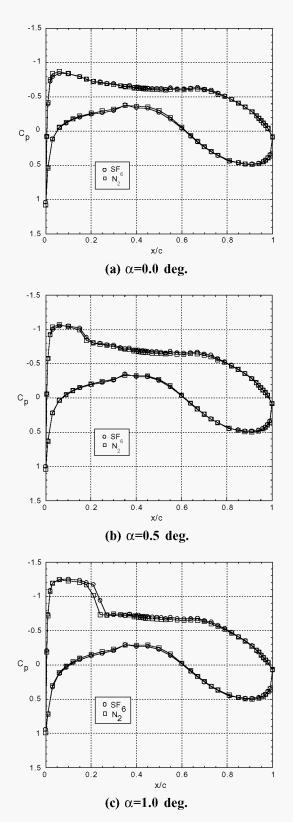
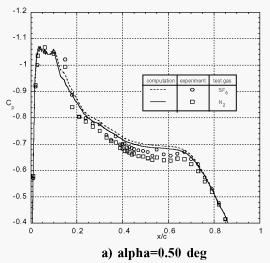


Fig. 7. Transonic similarity scaling applied to experimental results, Re_c =30x10 6 , $M_{\rm N2}$ =0.70, $M_{\rm SF6}$ =0.72



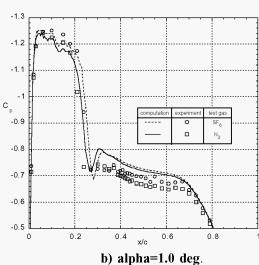


Fig. 8. Comparison of scaled experimental and computational results. $Re_c=30x10^6$, $M_{N2}=0.70$, $M_{SF6}=0.72$

Comments on Viscous Similarity

The fact that transonic similarity theory fails in the shock-boundary layer interaction region is not unexpected since it is based on small disturbance potential flow theory. Clearly, if the boundary layers are different in the two gases, then an inviscid transformation process will not account for this difference.

Calculations of the growth of the displacement thickness on the upper surface of the airfoil for both air and SF_6 (again, at the similarity Mach numbers of 0.70 for air and 0.72 for SF_6) indicate that the air boundary layer grows at a significantly greater rate (also see Anderson⁹). Figure 9 shows that for the present airfoil at roughly the

mid-chord location, the air displacement thickness is roughly 10% greater for air than for SF₆

It is tempting to suggest that this difference in δ^* , which results in an effective difference in the airfoil thickness between the two gases, may be accounted for by simply using an effective τ in the transonic similarity relations presented earlier. However, it is easy to show that if τ for the airfoil in air is increased by 10%, then the similarity Mach number for SF₆ increases from 0.72 to approximately 0.736, which would result in a further downstream movement of the SF₆ shock location relative to the nitrogen case, therefore increasing the disagreement between the two gases in the shock region.

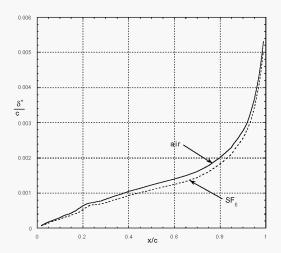


Fig. 9. Computation of displacement thickness growth in air and SF₆, Re_c= $30x10^6$, M_{SF6}=0.72, M_{air}=0.70.

The above calculation of the displacement thickness growth assumes that both boundary layers are fully turbulent. However, in reality, it is certainly possible that the transition process is different between the two gases. Unfortunately, transition was not measured in the present experiment, and one can only speculate regarding it's effect. Early transition for SF₆ could offset, or even reverse the results shown in figure 9, and/or it is possible that the response of the boundary layers to the adverse pressure gradient generated by the shock is somewhat different. In order to answer these and other questions regarding the viscous flow characteristics of heavy gas, detailed boundary layer investigations need to be conducted examining the viscous phenomena of transition, separation, and shock-boundary layer interaction.

Concluding Remarks

Previous computational studies showed that transonic similarity theory is successful in transforming transonic airfoil pressure distributions in SF₆ to equivalent results in air when the flows are primarily inviscid. However, this earlier work also indicated that the scaling is less successful when viscous effects are included. The current experimental results confirm the viscous computations and show that in the region of a shockboundary layer interaction the scaling always yields a shock location for heavy gas that is slightly downstream of the location for nitrogen (air). This mismatch is small, however, and the scaling may be adequate for many applications where only small regions of compressible flow are present. The interesting questions that remain to be answered for heavy gas flows are: how do the viscous processes of transition, separation, and shock-boundary layer interaction differ from air and what methods can be used to provide viscous similarity? These questions must remain the subject of future studies.

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